EXAMPLE 3–11 The Use of Generalized Charts

Determine the specific volume of refrigerant-134a at 1 MPa and 50°C, using (*a*) the ideal-gas equation of state and (*b*) the generalized compressibility chart. Compare the values obtained to the actual value of 0.021796 m^3/kg and determine the error involved in each case.

Solution The specific volume of refrigerant-134a is to be determined assuming ideal- and nonideal-gas behavior.

Analysis The gas constant, the critical pressure, and the critical temperature of refrigerant-134a are determined from Table A–1 to be

$$R = 0.0815 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$$

$$P_{\text{cr}} = 4.059 \text{ MPa}$$

$$T_{\text{cr}} = 374.2 \text{ K}$$

(*a*) The specific volume of refrigerant-134a under the ideal-gas assumption is

$$v = \frac{RT}{P} = \frac{(0.0815 \text{ kPa} \cdot \text{m}_3/\text{kg} \cdot \text{K})(323 \text{ K})}{1000 \text{ kPa}} = \mathbf{0.026325 \ m^3/kg}$$

Therefore, treating the refrigerant-134a vapor as an ideal gas would result in an error of (0.026325 − 0.021796)/0.021796 = **0.208**, or 20.8 percent in this case.

(*b*) To determine the correction factor *Z* from the compressibility chart, we first need to calculate the reduced pressure and temperature:

$$\left.\begin{aligned} P_R &= \frac{P}{P_{\text{cr}}} = \frac{1 \text{ MPa}}{4.059 \text{ MPa}} = 0.246 \\ T_R &= \frac{T}{T_{\text{cr}}} = \frac{323 \text{ K}}{374.2 \text{ K}} = 0.863 \end{aligned}\right\} \quad Z = 0.84$$

Thus

$$v = Zv_{\text{ideal}} = (0.84)(0.026325 \text{ m}^3/\text{kg}) = \mathbf{0.022113 \ m^3/kg}$$

Discussion The error in this result is less than **2 percent**. Therefore, in the absence of tabulated data, the generalized compressibility chart can be used with confidence.

EXAMPLE 3–13 **Different Methods of Evaluating Gas Pressure**

Predict the pressure of nitrogen gas at $T = 175$ K and $v = 0.00375$ m^3/kg on the basis of (*a*) the ideal-gas equation of state, (*b*) the van der Waals equation of state, (*c*) the Beattie-Bridgeman equation of state, and (*d*) the Benedict-Webb-Rubin equation of state. Compare the values obtained to the experimentally determined value of 10,000 kPa.

Solution The pressure of nitrogen gas is to be determined using four different equations of state.

Properties The gas constant of nitrogen gas is 0.2968 kPa · m^3/kg · K (Table A–1).

Analysis (*a*) Using the ideal-gas equation of state, the pressure is found to be

$$P = \frac{RT}{v} = \frac{(0.2968\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(175\ \text{K})}{0.00375\ \text{m}^3/\text{kg}} = \mathbf{13{,}851\ kPa}$$

which is in error by 38.5 percent.

(*b*) The van der Waals constants for nitrogen are determined from Eq. 3–23 to be

$$a = 0.175\ \text{m}^6\cdot\text{kPa/kg}^2$$

$$b = 0.00138\ \text{m}^3/\text{kg}$$

From Eq. 3–22,

$$P = \frac{RT}{v - b} - \frac{a}{v^2} = \mathbf{9471\ kPa}$$

which is in error by 5.3 percent.

(*c*) The constants in the Beattie-Bridgeman equation are determined from Table 3–4 to be

$$A = 102.29$$

$$B = 0.05378$$

$$c = 4.2 \times 10^4$$

Also, $\overline{v} = Mv = (28.013\ \text{kg/mol})(0.00375\ \text{m}^3/\text{kg}) = 0.10505\ \text{m}^3/\text{kmol}$. Substituting these values into Eq. 3–24, we obtain

$$P = \frac{R_u T}{\overline{v}^2}\left(1 - \frac{c}{\overline{v}T^3}\right)(\overline{v} + B) - \frac{A}{\overline{v}^2} = \mathbf{10{,}110\ kPa}$$

which is in error by 1.1 percent.

EXAMPLE 12–11 The Δh and Δs of Oxygen at High Pressures

Determine the enthalpy change and the entropy change of oxygen per unit mole as it undergoes a change of state from 220 K and 5 MPa to 300 K and 10 MPa (*a*) by assuming ideal-gas behavior and (*b*) by accounting for the deviation from ideal-gas behavior.

Solution Oxygen undergoes a process between two specified states. The enthalpy and entropy changes are to be determined by assuming ideal-gas behavior and by accounting for the deviation from ideal-gas behavior.

Analysis The critical temperature and pressure of oxygen are $T_{cr} = 154.8$ K and $P_{cr} = 5.08$ MPa (Table A–1), respectively. The oxygen remains above its critical temperature; therefore, it is in the gas phase, but its pressure is quite high. Therefore, the oxygen will deviate from ideal-gas behavior and should be treated as a real gas.

(*a*) If the O_2 is assumed to behave as an ideal gas, its enthalpy will depend on temperature only, and the enthalpy values at the initial and the final temperatures can be determined from the ideal-gas table of O_2 (Table A–19) at the specified temperatures:

$$\begin{aligned}(\bar{h}_2 - \bar{h}_1)_{\text{ideal}} &= \bar{h}_{2,\text{ideal}} - \bar{h}_{1,\text{ideal}} \\ &= (8736 - 6404)\ \text{kJ/kmol} \\ &= \mathbf{2332\ kJ/kmol}\end{aligned}$$

The entropy depends on both temperature and pressure even for ideal gases. Under the ideal-gas assumption, the entropy change of oxygen is determined from

$$(\bar{s}_2 - \bar{s}_1)_{\text{ideal}} = \bar{s}_2^\circ - \bar{s}_1^\circ - R_u \ln \frac{P_2}{P_1}$$

$$= (205.213 - 196.171)\ \text{kJ/kmol} \cdot \text{K} - (8.314\ \text{kJ/kmol} \cdot \text{K}) \ln \frac{10\ \text{MPa}}{5\ \text{MPa}}$$

$$= \mathbf{3.28\ kJ/kmol \cdot K}$$

(*b*) The deviation from the ideal-gas behavior can be accounted for by determining the enthalpy and entropy departures from the generalized charts at each state:

$$\left.\begin{aligned} T_{R_1} &= \frac{T_1}{T_{\text{cr}}} = \frac{220\ \text{K}}{154.8\ \text{K}} = 1.42 \\ P_{R_1} &= \frac{P_1}{P_{\text{cr}}} = \frac{5\ \text{MPa}}{5.08\ \text{MPa}} = 0.98 \end{aligned}\right\} Z_{h_1} = 0.53,\ Z_{s_1} = 0.25$$

and

$$\left.\begin{aligned} T_{R_2} &= \frac{T_2}{T_{\text{cr}}} = \frac{300\text{ K}}{154.8\text{ K}} = 1.94 \\ P_{R_2} &= \frac{P_2}{P_{\text{cr}}} = \frac{10\text{ MPa}}{5.08\text{ MPa}} = 1.97 \end{aligned}\right\} Z_{h_2} = 0.48, Z_{s_2} = 0.20$$

Then the enthalpy and entropy changes of oxygen during this process are determined by substituting the values above into Eqs. 12–58 and 12–63,

$$\begin{aligned} \overline{h}_2 - \overline{h}_1 &= (\overline{h}_2 - \overline{h}_1)_{\text{ideal}} - R_u T_{\text{cr}}(Z_{h_2} - Z_{h_1}) \\ &= 2332\text{ kJ/kmol} - (8.314\text{ kJ/kmol}\cdot\text{K})[154.8\text{ K}(0.48 - 0.53)] \\ &= \mathbf{2396\text{ kJ/kmol}} \end{aligned}$$

and

$$\begin{aligned} \overline{s}_2 - \overline{s}_1 &= (\overline{s}_2 - \overline{s}_1)_{\text{ideal}} - R_u(Z_{s_2} - Z_{s_1}) \\ &= 3.28\text{ kJ/kmol}\cdot\text{K} - (8.314\text{ kJ/kmol}\cdot\text{K})(0.20 - 0.25) \\ &= \mathbf{3.70\text{ kJ/kmol}\cdot\text{K}} \end{aligned}$$

Discussion Note that the ideal-gas assumption would underestimate the enthalpy change of the oxygen by 2.7 percent and the entropy change by 11.4 percent.